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Mathematical simulation of steady filtration with multivalued law

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Abstract

We consider a steady process of filtration of the incompressible high-viscosity fluid, following multivalued filtration law. Generalized statement of this problem is formulated in the form of mixed variational inequality with monotone operator and separable generally nondifferentiable functional in Hilbert space. To this problem the problem of finding the boundaries of limit-equilibrium unrecovered viscoplastic oil in multilayer beds can be reduced. We establish the properties of operator (inverse strong monotonicity, coerciveness) and functional (Lipschitz continuity, convexity) contained in this variational inequality. This makes it possible to apply the known results in the theory of monotone operators to prove the existence theorem. To solve the variational inequality, we suggest iterative method that does not require the inversion of the original operator. Each step of the iterative process can essentially be reduced to the solution of the boundary-value problem for the Laplace operator. The investigation of the convergence of the iterative process is performed by its reduction to the successive approximation method for finding a fixed point of some operator (the transition operator). We obtain a relationship between the solution of the original variational inequality and the components of the fixed point of this transition operator. We show that the transition operator is nonexpanding; moreover, we obtain an inequality stronger than the nonexpansion inequality. We also show that the transition operator is asymptotically regular. This permits one to prove the weak convergence of the successive approximations. This method was realized numerically. The numerical experiments made for the model problems confirmed the efficiency of the iterative method. It is must to be mentioned that the suggested methods permit one to find approximate values not only of the solution itself but also of its characteristics (for filtration problems, these are approximate values of the solution gradient and also the approximate values of filtration rate on the sets according to the points of multivalence in filtration law), which is very useful from the practical viewpoint.

Keywords

Inverse strongly monotone operator, Iterative method, Mathematical simulation, Nondifferentiable functional, Steady filtration, Variational inequality